

Parameterizing Ice Cloud Inhomogeneity and the Overlap of Inhomogeneities Using Cloud Radar Data

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ABSTRACT

Cloud variability on scales smaller than the gridbox size of numerical forecast and climate models is believed to be important in determining the radiative effects of clouds, and increasingly attempts are being made to parameterize these fluctuations in the radiation schemes of current models. In order to calculate the radiative effects of an inhomogeneous cloud, a model needs to know not only the degree of variability within a gridbox but also the degree to which the inhomogeneities in vertically adjacent levels are overlapped. In this paper these two parameters are derived for ice clouds from an 18-month midlatitude 94-GHz cloud radar dataset and parameterized in terms of horizontal gridbox size (d), the vertical shear of the horizontal wind (s), and the vertical position in the cloud. The vertical decorrelation length Δz_0 (i.e., the depth over which the correlation coefficient of either ice water content or optical extinction coefficient in separate vertical levels falls to e^{-1}) is found to be well represented in the mean by $\log_{10}\Delta z_0 = 0.3 \log_{10}d - 0.031s - 0.315$, where Δz_0 and d are in kilometers and s is in meters per second per kilometer. As expected, higher shear results in more rapid decorrelation, although the rms deviation from this expression is around a factor of 2.5. It is found that the probability distribution of ice water content within a gridbox is usually well represented by a lognormal or gamma distribution. The fractional variance in ice water content (f_{IWC}) may be expressed to within a factor of 2 by $\log_{10}f_{\text{IWC}} = 0.3 \log_{10}d - 0.04s - 0.93$, valid for $d < 60$ km, above which f_{IWC} is constant with increasing d . The expression for the fractional variance of visible extinction coefficient is the same except with the -0.93 term replaced by -0.96 . The s dependence indicates a tendency for increased shear to result in *decreased* cloud variability. This can be explained by the presence of ice fallstreaks in a sheared flow: a parcel of air in the middle of a cloud is alternately fed from above by ice-rich and ice-poor air, resulting in a homogenization of the layer at a rate dependent on the shear. A more complicated formula is derived to express the dependence of f_{IWC} on the vertical position within the cloud; it is found that fractional variance tends to be largest at cloud top and decreases into the interior before increasing again in the lowest third of the cloud. Thicker clouds tend to have lower fractional variance. No significant dependence on temperature or absolute altitude was found for either f_{IWC} or Δz_0 .

1. Introduction

The importance of ice clouds on the earth's radiation budget is well recognized (Liou 1986), and increasingly attention is focusing on the role of small-scale cloud variability and how to represent it in models (e.g., Liou and Rao 1996; Donner et al. 1997). Most current forecast and climate models represent the fractional cloudiness of a model gridbox either diagnostically from the relative humidity (e.g., Slingo 1987; Smith 1990; Lohmann and Roeckner 1996), or as a prognostic variable (Tiedtke 1993). Recently, Tompkins (2002) proposed a more advanced scheme that also carries the variance and skewness of the total water content (vapor and condensed phases) as prognostic variables; cloud fraction and condensed water content are then determined by

splitting the probability distribution function (PDF) of total water into cloudy and cloud-free components based on the saturation specific humidity. However, the implied distribution of condensed water content in the cloudy part of the box is not currently used in the other parameterizations of these models, which usually treat clouds as horizontally homogeneous.

For boundary layer clouds, the primary effect of cloud inhomogeneity is to reduce the albedo relative to that computed using the plane-parallel approximation. In an attempt to correct this problem in the model of the European Centre for Medium-Range Weather Forecasts (ECMWF), Tiedtke (1996) used the findings of Cahalan et al. (1994) and simply multiplied water content by 0.7 in the calculation of albedo. Barker et al. (1996) proposed a more sophisticated method, which involves representing the distribution of optical depth by a gamma distribution function, and validated it using high-resolution satellite imagery. However, if models do not include the effects of cloud variability in all their parameterizations, then significant microphysical and

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thermodynamical biases can also occur (Larson et al. 2001).

This paper addresses the issue of subgrid-scale inhomogeneity in ice clouds, which has generally received much less attention than liquid cloud inhomogeneity. Ice clouds are generally quite deep, occupying several vertical gridboxes, so a single path-integrated satellite estimate of cloud structure is unsatisfactory. Instead, we appeal to the new technology of cloud radar, which provides range-resolved data of cloud structure from the ground and, unlike instruments that operate at optical wavelengths, suffers virtually no attenuation in ice clouds. In the last few years the ability of these instruments to detect cloud boundaries has been exploited to evaluate model representation of cloud occurrence (Mace et al. 1998), cloud fraction (Hogan et al. 2001), and cloud overlap (Hogan and Illingworth 2000). The ability of radar to estimate cloud variables such as ice water content (IWC) was used by Pomroy and Illingworth (2000) and Fu et al. (2000) to address the specific problem of the emissivity bias in models due to ice cloud variability.

Our intention here is to use 94-GHz cloud radar to provide the essential data on ice cloud variability that is needed by the newer model parameterizations that attempt to represent subgrid-scale cloud structure. This information could be used either as the basis for a diagnostic parameterization for the variance of ice water content, or to evaluate schemes that carry variance as a prognostic variable, such as that of Tompkins (2002). Brown et al. (1995) estimated that IWC could be derived from radar reflectivity factor (Z) with an error of around a factor of 2 (the typical range of IWC we wish to measure being around three orders of magnitude). Liquid water clouds, on the other hand, often contain low concentrations of “drizzle” droplets, which can dominate Z while containing negligible liquid water, with the result that Z is often essentially unrelated to stratocumulus liquid water content (Fox and Illingworth 1997). We therefore restrict our analysis to ice clouds.

The first parameter to be estimated is the fractional variance of ice water content within a model gridbox, which we define as

$$f_{\text{IWC}} = (\sigma_{\text{IWC}}/\overline{\text{IWC}})^2, \quad (1)$$

where σ denotes standard deviation. Similarly, the fractional variance of visible extinction coefficient α is defined as

$$f_{\alpha} = (\sigma_{\alpha}/\overline{\alpha})^2. \quad (2)$$

The relationship between fractional variance and the parameters used to describe gamma and lognormal distributions is outlined in section 2. Then in section 3, aircraft data taken during the European Cloud Radiation Experiment (EUCREX) are used to derive the appropriate functions relating radar reflectivity to IWC and α .

The other important parameter, in addition to fractional variance, is the vertical decorrelation length Δz_0 ,

which is the depth over which the correlation coefficient of IWC in separate vertical layers falls to e^{-1} . This is important because rapid decorrelation makes a cloud much more effective at blocking outgoing thermal radiation and reflecting incoming visible radiation, compared with the case of the inhomogeneities being stacked above each other, even though the total water content may be the same. Hogan and Illingworth (2000) used radar data to derive a similar parameter for describing “cloud overlap” (Morcrette and Fouquart 1986; Liang and Wang 1997) for use in current models, and it should be stressed that the two are not necessarily the same; the decorrelation length of Hogan and Illingworth (2000) was derived considering cloud boundaries alone, while here we are considering the overlap characteristics of the in-cloud fluctuations of water content. Nonetheless, the two extremes of $\Delta z_0 = 0$ and $\Delta z_0 = \infty$ can be thought of as being analogous to the “random” and “maximum” overlap assumptions made in models, respectively.

In section 4 we present radar observations of a thick ice cloud and demonstrate the effects of gridbox size and wind shear on the parameters f_{IWC} and Δz_0 . The suitability of gamma and lognormal distributions for fitting the observed PDFs is investigated. Then in section 5, 18 months of near-continuous data taken by the same instrument are analyzed to obtain empirical relationships for these parameters.

2. Analytical probability distribution functions

Here we show how fractional variance is related to the parameters describing analytical PDFs used to represent the distribution of IWC and α within a gridbox. Barker et al. (1996) used a gamma distribution to represent the optical depth (τ) distribution of boundary layer stratocumulus and cumulus as inferred from a satellite:

$$p(\tau) = \frac{1}{\Gamma(\nu)} \left(\frac{\nu}{\bar{\tau}}\right)^{\nu} \tau^{\nu-1} \exp\left(-\frac{\nu\tau}{\bar{\tau}}\right), \quad (3)$$

where $\nu = (\bar{\tau}/\sigma_{\tau})^2$ and $\Gamma(\nu)$ denotes the gamma function. If the same distribution is used for IWC instead of τ , then we have $\nu = f_{\text{IWC}}^{-1}$.

A lognormal distribution for IWC has the form

$$p(\text{IWC}) = \frac{1}{(2\pi)^{1/2}\sigma_0\text{IWC}} \exp\left[-\frac{\ln(\text{IWC}/\text{IWC}_0)^2}{2\sigma_0^2}\right]. \quad (4)$$

The parameters of the distribution, IWC_0 and σ_0 , may be defined in terms of $\overline{\text{IWC}}$ and f_{IWC} as follows:

$$\text{IWC}_0 = \overline{\text{IWC}}(f_{\text{IWC}} + 1)^{-1/2}; \quad (5)$$

$$\sigma_0^2 = \ln(f_{\text{IWC}} + 1). \quad (6)$$

Thus the results in this paper may be easily compared to other papers that express cloud variability in a different way. The suitability of gamma and lognormal

distributions for parameterizing the IWC distribution of real clouds is studied in section 4c.

3. Deriving relationships from the EUCREX dataset

a. Accounting for the small ice particles

To determine the appropriate expressions relating radar reflectivity to IWC and α we follow Brown et al. (1995) and use size spectra recorded by the Met Office C-130 aircraft in midlatitude ice clouds during EUCREX. The dataset consists of over 10 000 5-s averaged size spectra measured by the 2D cloud and precipitation probes, which cover the diameter range 25 to 6400 μm . However, the 2D cloud probe (2D-C) is known to be somewhat unreliable for measuring particles smaller than 100 μm (Heymsfield and Baumgardner 1985). Francis et al. (1998) used replicator measurements of the small crystals taken during a few runs of EUCREX to suggest a correction for the whole dataset, which involves fitting a gamma distribution to the lower end of the measured size distribution. The gamma distribution was constrained to have a modal diameter of 6 μm , to have the same concentration of particles of diameter 100 μm as measured by the 2D-C, but to have 5 times more particles of diameter 25 μm . Note that diameter here refers to the diameter of a circle with the same cross-sectional area as the particle image measured by the probe. The intention of Francis et al. was to produce a “pessimistic” estimate of the undercounting of the small particles, and in the example plotted in their paper, the replicator size spectrum for a diameter of 25 μm was only 3 to 3.5 times higher than the concentration measured in the first bin of the 2D-C. The one or two other runs for which replicator data were available showed a much smaller difference between the two spectra (P. N. Francis 2002, personal communication). McFarquhar and Heymsfield (1997) compared the ice water content contained in the particles with a maximum dimension smaller than 90 μm (hereafter $\text{IWC}_{<90}$) as measured by the 2D-C and the Video Ice Particle Sampler (VIPS). From Fig. 20a of their paper, we estimate that the 2D-C probe underestimated $\text{IWC}_{<90}$ by a factor of 2.5, on average, although there was considerable scatter in their data and on some occasions the 2D-C measured higher values than VIPS.

With these considerations in mind, we fit a gamma distribution to the small particles in the same manner as Francis et al. (1998), but constrain the fitted distribution to be only a factor of 2 higher than the measured distribution for 25- μm particles. This increases $\text{IWC}_{<90}$ by around a factor of 2.5, in agreement with the findings of McFarquhar and Heymsfield (1997).

From these modified distributions we calculate IWC using the mass–area relationship of Francis et al. (1998), while α is defined to be simply twice the total cross-sectional area of particles per unit volume. Radar re-

flectivity at 94 GHz is calculated in the same manner as Brown et al. (1995) and Hogan and Illingworth (1999); the crystals are approximated as homogeneous ice–air spheres with a diameter D_m equal to the mean of the maximum particle dimensions measured parallel and perpendicular to the probe photodiode array. Their density is determined from the mass– D_m relationship of Brown and Francis (1995), and Mie theory is applied. It should be noted that D_m is systematically larger than the “equivalent area” diameter, but is believed to better represent the extremities of the particle, which are important when the scattering at 94 GHz departs from the Rayleigh approximation.

b. The standard deviation (SD) line

We wish to derive relationships of the form

$$X = cZ^b, \quad (7)$$

where X is either IWC or α , and Z is in $\text{mm}^6 \text{m}^{-3}$. Since in this paper we are considering the fractional variance of IWC and α , only the exponent b in (7) is important. The Δz_0 parameter is also completely insensitive to the c coefficient. Therefore, the results are unaffected by any error in radar calibration.

Numerous Z –IWC power-law relationships exist in the literature, and are typically derived by performing a linear regression between the two parameters in logarithmic space. The slope of the regression line is then equal to b . This approach produces the best estimate of IWC for any given measurement of Z , but in this study we are really interested in obtaining the best estimate of the fractional variance of IWC given a measurement of the fractional variance of Z . Since fractional variance is equivalent to the variance of the logarithm, we require our best-fit line to have a slope $b = \sigma_{\log\text{IWC}}/\sigma_{\log Z}$, where σ denotes standard deviation. However, according to statistical theory, the slope of the regression line is actually $r\sigma_{\log\text{IWC}}/\sigma_{\log Z}$, where r is the correlation coefficient between $\log\text{IWC}$ and $\log Z$. This simply expresses the definition of the correlation coefficient: with each one-standard-deviation increase in one of the variables, there is an increase of only r standard deviations in the other, on the average. Invariably, r is significantly less than unity, so relationships derived by linear regression in logarithmic space will tend to result in an underestimate of f_{IWC} by a factor of r^2 . We therefore use the *SD line*¹ rather than the regression line as the basis for relationships between Z and other parameters.

A further consideration is that fits to an entire dataset include the variance associated with changing altitude and temperature, whereas each retrieved value of f_{IWC} in this study will correspond to one particular altitude and temperature. The SD line is therefore calculated

¹ In statistics, the “standard deviation line,” or SD line, for abscissa x and ordinate y is defined to have a slope of σ_y/σ_x and pass through the point (\bar{x}, \bar{y}) .

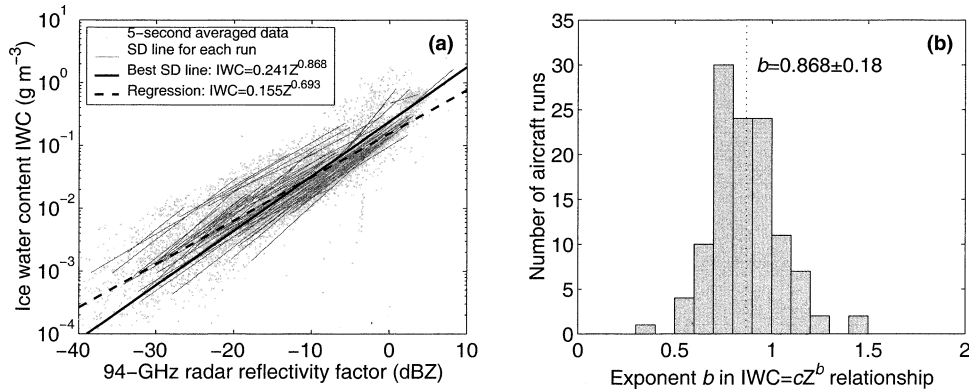


FIG. 1. Retrieval of Z -IWC power law from EUCREX data. (a) The light gray points indicate the individual 5-s samples, and the thin solid lines indicate the SD lines for each of the 115 horizontal aircraft runs in EUCREX with their length indicating ± 1 std dev. (b) A histogram of the slopes of these lines; the “best” SD line, with a slope equal to the mean of the slopes of the individual runs, is shown by the thick solid line in (a). The thick dashed line shows the regression, and indicates the difference in slope that can occur when the parameters are not perfectly correlated. Note that Z is plotted in logarithmic dBZ units, where $Z_{\text{dBZ}} = 10 \log_{10}(Z_{\text{mm}^6 \text{m}^{-3}})$.

separately for each horizontal aircraft run in the EUCREX dataset.

The thin solid lines in Fig. 1a depict the SD lines for each of the 115 aircraft runs of EUCREX (two of the runs consisted of fewer than ten 5-s samples and were rejected). The “best” SD line, shown by the thick solid line, passes through the point of means—that is, $(\overline{\log IWC}, \overline{\log Z})$ —and has a slope equal to the mean of the slopes of the individual runs. Its formula is

$$IWC = 0.241Z^{0.868}, \quad (8)$$

where Z is in $\text{mm}^6 \text{m}^{-3}$ and IWC is in g m^{-3} . The range of exponents in the individual runs is indicated in Fig. 1b. The standard deviation of 0.18 ($\pm 21\%$) suggests a random rms error in f_{IWC} retrieved using 94-GHz radar of $\pm 42\%$. The regression line (shown by the thick dashed line in Fig. 1a) has a significantly smaller slope of 0.693, and is much closer to values previously reported in the literature (see Liu and Illingworth 2000). The effect of not performing the small-particle correction described in section 3a is to increase the slope of

the regression line by 10%, although the slope of the best SD line in (8) is changed by only 2%.

It should be stressed that (8) is *not* suitable for retrieving a best estimate of the absolute values of IWC, nor should it be used at frequencies other than 94 GHz. It may be used in the retrieval of Δz_0 , however, since Δz_0 is very insensitive to the exact form of the Z -IWC relationship, or indeed, to which variable is being correlated.

The analogous relationship for visible extinction coefficient is derived in exactly the same way (see Fig. 2):

$$\alpha = 0.00691Z^{0.841} \text{ m}^{-1}. \quad (9)$$

The error in retrieved f_a is around $\pm 45\%$. Due to the lower correlation between Z and α , the slope of the regression line is significantly lower at 0.558. It may appear counterintuitive that the exponents in (8) and (9) are so similar when IWC and α represent different moments of the size distribution, but this is due to several factors. First, IWC should not be regarded as the third

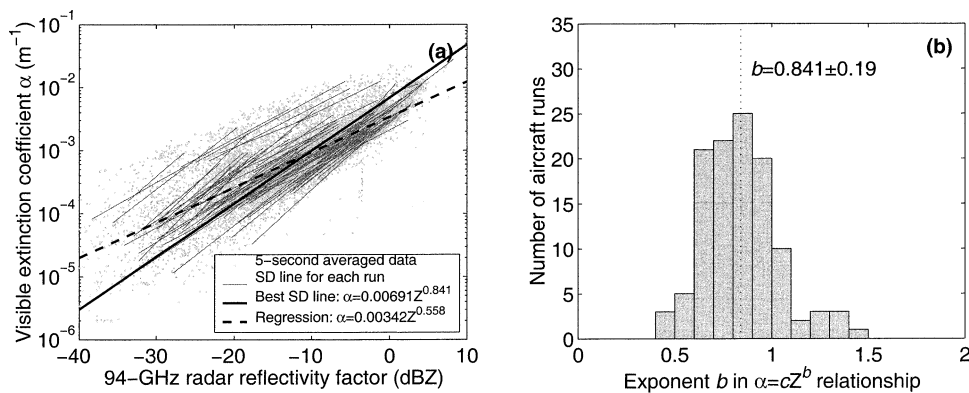


FIG. 2. As in Fig. 1, but for visible extinction coefficient α .

moment of the size distribution, but a moment between the second and third (and therefore closer to α), because of the tendency for ice particle density to decrease with size (Brown and Francis 1995; Francis et al. 1998). Second, much of the variability in the various moments of the distribution is not due to changes in mean particle size, but to changes in total number concentration that affect each moment equally. Moreover, by calculating the SD lines separately for each horizontal aircraft run, the contribution of particle size variability is minimized since size varies predominantly in the vertical.

It is straightforward to deduce how the exponent will affect our retrieval of f_{IWC} : (8) implies that the fractional standard deviation of IWC within a gridbox will be close to 0.868 times that of Z itself. Therefore, to change to a new exponent in (8) simply involves scaling any derived formulas for f_{IWC} by the square of the ratio of the new and the old exponents. Likewise, the fractional variance of other bulk cloud parameters that can be related to Z by a power law may be derived simply from f_{IWC} . Hence, from the ratio of exponents in (9) and (8) it can be seen that $f_{\alpha} = 0.939f_{\text{IWC}}$.

It should be mentioned that there is a slight dependence of the slope of the SD lines on temperature; the correlation between the slope b of the Z -IWC relationship and temperature is 0.40, while for the Z - α relationship it is 0.18. The reason for the correlation is that at higher temperatures the ice particles tend to be larger. In the Rayleigh scattering regime this would be sufficient only to change the c coefficient in (7), but at high frequencies such as 94 GHz, the largest particles scatter outside the Rayleigh regime, which changes the slope as well. Because of the inherent uncertainties in our representation of the non-Rayleigh effects, and the low correlation between b and temperature, we choose not to include this effect in our relationships.

4. 27 December 1999 case study

a. Fallstreak structure

Figure 3a shows a 4-h time–height section of Z measured in thick ice cloud by the vertically pointing 94-GHz Galileo radar, located at Chilbolton, England (51.145°N, 1.437°W). The data were taken on 27 December 1999 and the resolution is 30 s and 60 m. A very distinct fallstreak structure is evident throughout the depth of the cloud. Above around 7 km, the cloud inhomogeneities associated with the fallstreaks appear to be close to vertically aligned, while below this height they become much more slanted. This structure can be understood on examination of the 1100 UTC wind profile in Fig. 3c (obtained from the mesoscale version of the Met Office Unified Model), which shows a strong zonal jet exceeding 60 m s⁻¹ between 7- and 8-km altitude where the shear was less than 5 m s⁻¹ km⁻¹. Immediately below 6.9 km, however, the shear exceeds

20 m s⁻¹ km⁻¹. At an altitude of 5 to 6 km, the gradient of the fallstreaks in the time–height image is between 10 and 13 min km⁻¹; from the model winds this implies that the fallstreaks had a horizontal–vertical aspect ratio of 25 to 30. The wind vector difference of around 30 m s⁻¹ between the jet and an altitude of 5.5 km is therefore consistent with an ice fall speed of around 1 m s⁻¹.

This simple comparison between model and observations demonstrates qualitatively the importance of shear in determining the orientation of fallstreaks, and hence the correlation of horizontal cloud inhomogeneities from one vertical level to the next.

b. Fractional variance of IWC

We next consider how the inhomogeneity evident in this cloud would appear in a model. Reflectivity factor is first averaged to 0.3 km in the vertical, imitating the highest resolutions that are expected to be achieved by operational mesoscale models in the next few years. Ice water content is then calculated using (8) and is shown in a linear scale in Fig. 3b. We consider a horizontal gridbox size of 50 km by using the model wind speeds to select periods of time between 1000 UTC and 1400 UTC equivalent to this distance. Only fully cloudy boxes are considered, because a model implementing a parameterization for cloud variability would doubtless have also implemented an ordinary cloud fraction scheme to cope with partially cloudy gridboxes. It should be noted that the direct correspondence between time and space assumed here (and throughout the paper) may “bias in” variations due to cloud temporal evolution, especially in low wind speed conditions or for larger gridbox sizes. However, many models only call their radiation schemes every 3 h, in which time considerable evolution can occur. It is therefore possible that too little cloud evolution is being included in our results, rather than too much.

A value for f_{IWC} is calculated for each 50 km × 0.3 km box, and Fig. 3d shows the mean and standard deviation at each height. Because of the increase in wind speed with height, the data that make up the 50-km samples are around 3 times better resolved at the bottom of the cloud than the top. This should not affect the results since the variability occurs predominately at scales much larger than the 30-s resolution of the data. It can be seen that mean f_{IWC} decreases from 0.8 near cloud top to 0.1 at 5.7 km. Looking at Fig. 3a, it seems that after formation at cloud top the fallstreaks remained relatively coherent in the first 1–2 km where the vertical wind shear was low. Then, in the region of much higher shear below around 7 km, much of the fine structure was lost, particularly after 1200 UTC, with an ensuing fall in f_{IWC} . This can be explained by considering a parcel in the middle of the sheared region; the differential velocity with the layer above would result in it being alternately fed by ice-rich and ice-poor air, resulting in a homogenization of the horizontal ice dis-

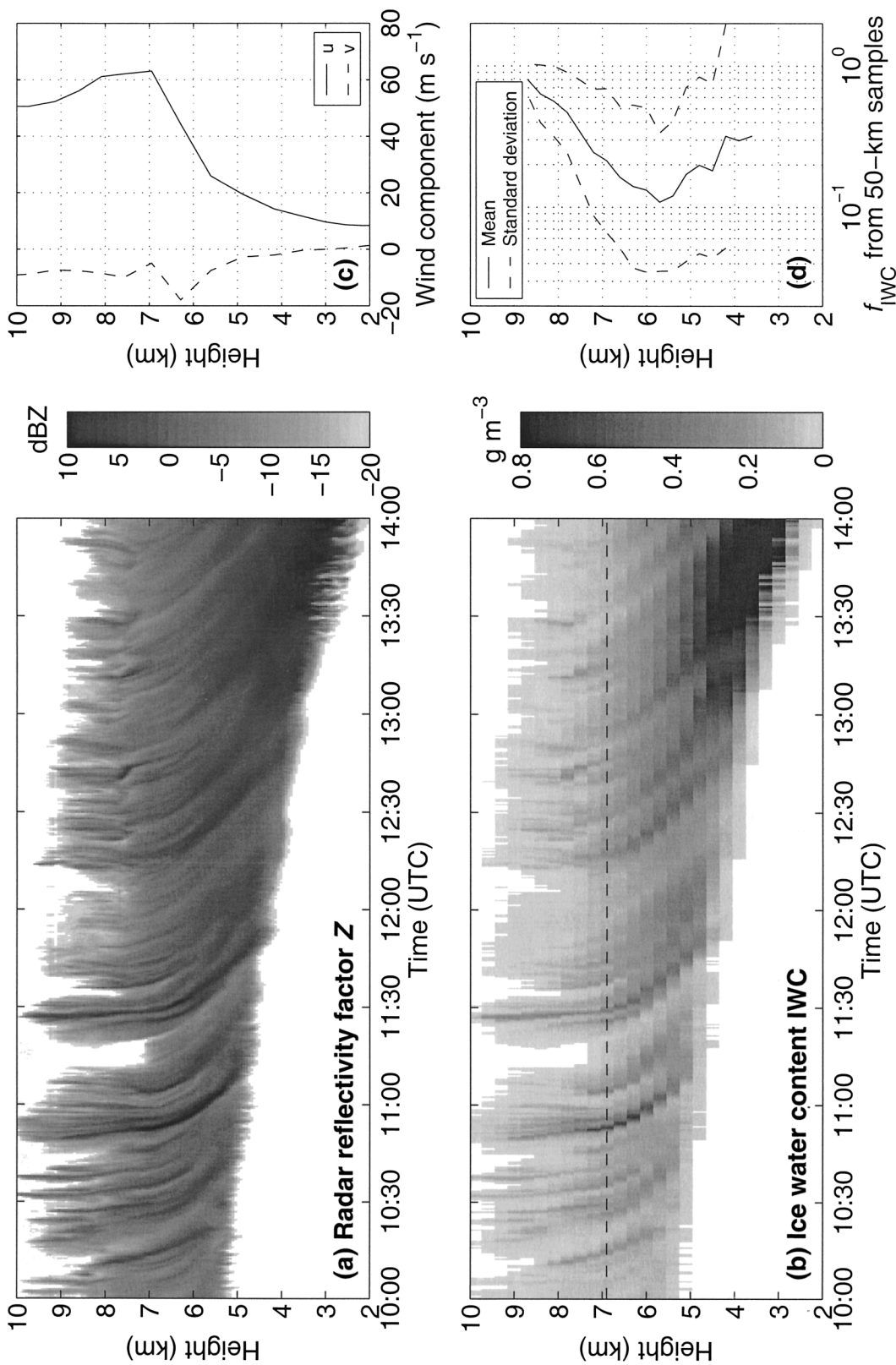


FIG. 3. (a) Radar reflectivity factor measured by the Galileo radar on 27 Dec 1999, and (b) IWC calculated from it in 0.3-km layers using the Z-IWC relationship in (8). The dashed line at an altitude of 6.9 km in (b) shows the division between the low shear and high shear cases considered in the text and in Fig. 6. (c) The zonal (u) and meridional (v) wind components at 1100 UTC over Chilbolton according to the mesoscale version of the Met Office Unified Model. (d) The mean fractional variance of IWC at each height, calculated from 50-km horizontal samples of cloud, and using the wind speed from (c) to determine the appropriate sample time at each height.

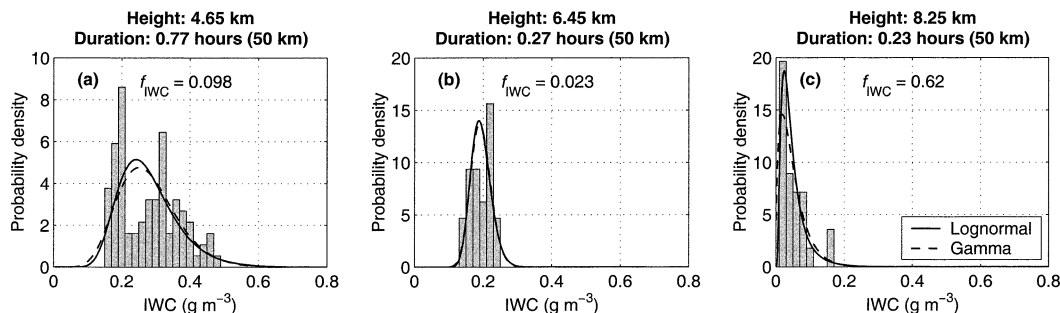


FIG. 4. PDFs of ice water content in three boxes measuring 50 km in the horizontal and 0.3 km in the vertical, taken from the data shown in Fig. 3 at altitudes of (a) 4.65, (b) 6.45, and (c) 8.25 km. The dwell times required to sample the equivalent of 50 km were determined from the wind speed over Chilbolton as reported by the mesoscale version of the Met Office model, and each box was centered on 1230 UTC. Overlaid on each panel are lognormal and gamma distributions fitted such that they have the same mean and variance as the data. The f_{IWC} parameter [defined in Eq.(1)] is also indicated.

tribution at a rate dependent on the magnitude of the shear.

Below 5.7 km, the mean f_{IWC} increases again toward cloud base. This is due to the base of the cloud becoming steadily lower during the 4-h period, with the result that significant horizontal gradients are present across the gridboxes near cloud base. Also, strong small-scale variability is present at cloud base after around 1330 UTC. This is most likely to be instability generated by the latent heat release associated with sublimation, and is not uncommon at the base of thick ice clouds.

The distinctive behavior evident in Fig. 3d suggests that f_{IWC} could be parameterized effectively in terms of wind shear and distance from cloud top or cloud base. However, the standard deviation of f_{IWC} in Fig. 3d increases from $\pm 25\%$ near cloud top to $+300\%/-75\%$ below 5 km. So, even in a single cloud with similar shear conditions persisting throughout the period that it is observed, different f_{IWC} values are able to evolve. This limits the ability of a parameterization to predict f_{IWC} precisely. Indeed, we should conclude that the cloud fractional variance in *individual* model gridboxes (particularly below cloud top) is impossible to predict accurately based on the information available to large-scale models, but that the *mean* fractional variance over many realizations may be possible to predict accurately. For the purposes of climate modeling this is probably sufficient, especially given that most current models effectively assume that $f_{IWC} = 0$ in all ice clouds. The same reasoning holds for current cloud overlap schemes; the overlap behavior of individual clouds can vary greatly, but if models can capture the mean overlap characteristics of clouds, then the corresponding radiative fluxes should be unbiased.

The range of f_{IWC} values observed in this case study is comparable to values that may be derived from the results of other researchers. Smith and DelGenio (2001) plotted aircraft measurements of σ_{IWC} against \overline{IWC} in cirrus for runs between 50 and 300 km in length. Figure 7a of their paper indicates values of f_{IWC} ranging be-

tween 0.09 and 1.5, with a median of around 0.5. Presumably, the larger values corresponded to the longer aircraft runs. Barker et al. (1996) examined the distribution of τ in $60 \text{ km} \times 60 \text{ km}$ regions of boundary layer cloud observed by satellite, and for overcast stratocumulus reported ν [defined in Eq. (3)] to be 7.98 ± 6.29 , equivalent to a fractional variance of optical depth f_τ of 0.13 ± 0.10 . For broken stratocumulus they found $f_\tau = 0.83 \pm 0.42$. It is interesting that this range of values for f_τ is similar to the 0.1–0.8 range for f_{IWC} found in Fig. 3c, despite the fact that quite different parameters and cloud types are being compared.

c. Probability distribution functions

In this section, the subgrid-scale distribution of ice water content is investigated. Figure 4 shows PDFs of IWC measured in three 50-km boxes at different heights in the cloud, each centered on 1230 UTC. As before the model winds have been used to determine the appropriate sample time. The values of f_{IWC} are shown in each panel. Overlaid on each measured PDF are lognormal and gamma distribution functions, fitted such that they have the same mean and variance as the original data. In these cases there is little difference between the two functions and both fit the data reasonably well, although there is some bimodality evident at 4.65 km.

Figure 5 shows PDFs at the same heights as in Fig. 4, but using boxes with a horizontal size of 250 km. The occurrence of larger values of f_{IWC} indicates the need to include gridbox size in any parameterization for cloud variability. Again, the PDFs at 6.45 and 8.25 km are well represented by both lognormal and gamma distributions, but this time the PDF at 4.65 is very poorly represented by either. A “top hat” function would probably describe the distribution better. The reason is that most of the variance at this altitude is not due to small-scale fluctuations but to a trend in IWC across the gridbox, from zero (i.e., cloud-free air) at the beginning of the period to 0.7 g m^{-3} in the thick fallstreaks at the

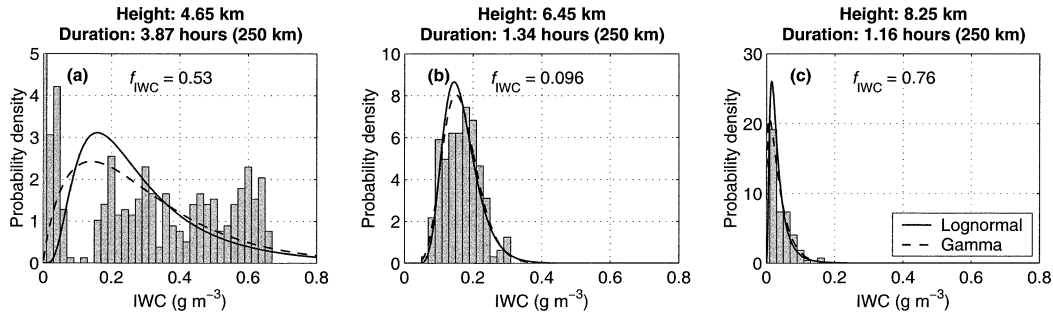


FIG. 5. As in Fig. 4, but for boxes measuring 250 km in the horizontal.

end. A model may be able to use the resolved horizontal gradient in IWC to determine how much variance within each box is due to such a trend, but this is only likely to be necessary for relatively low resolution models.

d. Vertical correlation of ice water content

The next step is to look at the correlation of the horizontal structure with height. The IWC between 1000 UTC and 1400 UTC is analyzed separately in the height ranges 6.9 to 8.9 km (the “low shear” case) and 4.9 to 6.9 km (the “high shear” case). A model horizontal gridbox size of 50 km is simulated by dividing the data above 6.9 km (where the mean wind speed was 63 m s^{-1}) into 13-min boxes and the data below this height (where the mean wind speed was 38 m s^{-1}) into 22-min boxes.

In each column of boxes, the correlation coefficient ρ is computed for the IWC of each level with each other level, neglecting partially cloudy boxes, and with no “double counting.” The circles in Fig. 6 depict the mean ρ versus vertical separation Δz for the low shear and high shear regions. In both cases the correlation of the horizontal structure decreases rapidly with Δz , and for

$\Delta z > 1 \text{ km}$ it is essentially uncorrelated. The difference in results is negligible if the correlation is performed on $\ln(\text{IWC})$ instead of IWC. The solid lines in Fig. 6 are simple inverse exponential expressions of the form

$$\bar{\rho} = \exp\left(-\frac{\Delta z}{\Delta z_0}\right), \quad (10)$$

with Δz_0 chosen to give the correct $\bar{\rho}$ for adjacent layers, 0.3 km apart. The corresponding decorrelation lengths Δz_0 are 0.68 and 0.35 km for the low and high shear cases, respectively. Hence, as expected, high shear produces a much more rapid decorrelation with height.

This analysis is similar to that performed by Hogan and Illingworth (2000) in their determination of an e-folding lengthscale over which the overlap of clouds in a partially cloudy gridbox becomes random, but there are some interesting differences. First, they reported values in the range 1.40–2.93 km, depending on the resolution of the gridbox, whereas our values are a factor of 4 smaller. This presumably reflects the larger horizontal and vertical scales that cloud boundaries exist on compared with the scales of subcloud variability. Second, they reported that several days of data were nec-

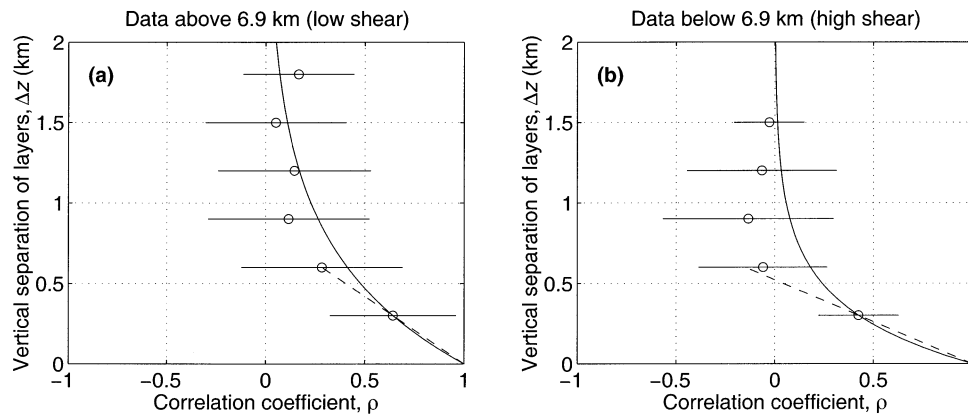


FIG. 6. The circles depict the mean correlation coefficient of the ice water content in $50 \text{ km} \times 0.3 \text{ km}$ boxes vs vertical separation. (a) The results for the data between 6.9 and 8.9 km in Fig. 3 (low shear case). (b) The results for the data between 4.9 and 6.9 km (high shear case). The error bars indicate the std dev of the raw data. The solid and dashed lines show exponential and linear fits to the first point at $\Delta z = 0.3 \text{ km}$, as discussed in the text.

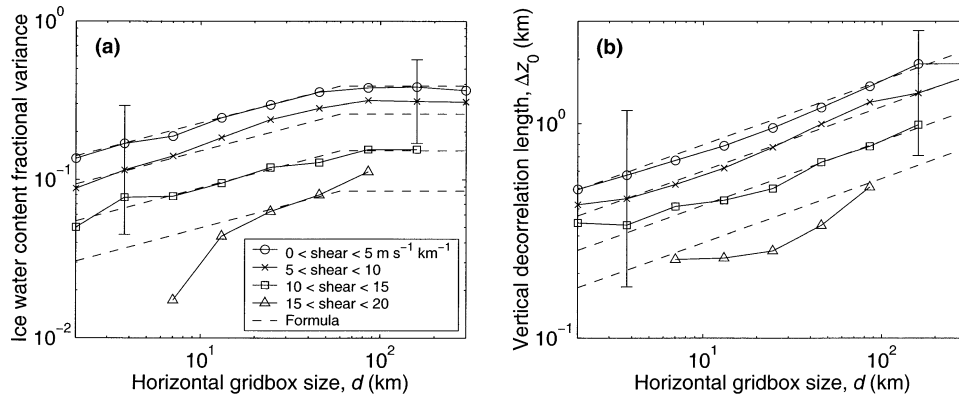


FIG. 7. The parameters (a) f_{IWC} and (b) Δz_0 (averaged from 18 months of cloud radar data) vs horizontal gridbox size and the magnitude of the vertical wind shear s . The dashed lines show the analytical fits given in (11) and (12). The error bars indicate the range of raw values from which the means were calculated, for shear in the range 5–10 m s⁻¹ km⁻¹. The spread is similar for other values of shear.

essary to obtain something approaching an inverse exponential relationship, while as little as 4 h of data seem to be sufficient here.

At this point we should consider whether an inverse exponential is really the best way of characterizing the relationship between correlation coefficient and vertical separation. Although the inverse exponential falls within all the error bars of the observations (indicating the range of ρ that was actually measured), the biggest difference between it and the mean ρ occurs for a vertical separation of 0.6 km, where ρ was significantly less than that indicated by the inverse exponential. In fact, it seems that the first 0.6 km would be much better characterized by a *linear* decrease of ρ with Δz (indicated by the dashed lines in Fig. 6). This is presumably because the decorrelation is due to fallstreaks, coherent over several kilometers in the vertical, which pass through the columns of boxes at an angle, rather than being due to some genuinely random process. It is interesting that the same phenomenon appears to be present to a small extent in the randomization of cloud overlap (Hogan and Illingworth 2000, Fig. 4). Despite this behavior, the results presented in the remainder of this paper are in terms of decorrelation length because of its appeal in conceptualizing the phenomenon of cloud decorrelation in the vertical, and because it is independent of the vertical resolution under consideration. The reader should be aware, however, that it is calculated simply from the correlation coefficient between layers 0.3 km apart.

5. Analysis of 18 months of radar data

We now extend the analysis performed in the last section to the near-continuous observations taken at Chilbolton by the 94-GHz Galileo cloud radar between May 1999 and October 2000. Again, the derived IWC data are used at a resolution of 0.3 km in the vertical and 30 s temporally, and only fully cloudy boxes are

considered. The data are analyzed in boxes of duration varying between 5 min and 3 h. Because of the possibility of variable attenuation by rain, which would tend to increase the correlation between the layers above, data taken by a drop-counting rain gauge at Chilbolton were used to reject whole columns of data from the analysis whenever rain was detected. Attenuation by the ice clouds themselves is negligible (Hogan and Illingworth 1999). For each box the fractional variance of ice water content f_{IWC} is computed, and for every pair of vertically adjacent boxes, we calculate the correlation coefficient ρ .

These values are then combined with data from the Unified Model over Chilbolton, which consist of the 6-hourly analyses and the intervening hourly forecasts. The model wind speeds are used to convert the 15 different temporal resolutions into horizontal resolutions. Clouds warmer than -5°C are rejected to prevent contamination of the results by the melting layer. The magnitude of the wind shear is extracted at the height of every f_{IWC} and ρ observation.

We first classify the resulting 221 659 data points by horizontal box size d and wind shear s . Nine d classes are used, the mean sizes of which are spaced logarithmically between 2 and 300 km. The data are then further divided according to s into 5 m s⁻¹ km⁻¹ intervals, and mean f_{IWC} is calculated in each class. The decorrelation length is calculated from the median ρ in each class.

These values are plotted in log-log space in Fig. 7 as a function of d and s . Means calculated from fewer than 100 data points have not been plotted because of doubts over their statistical representativity. These points represent means of very scattered data as indicated by the error bars; the standard deviation of f_{IWC} is around +100% to -50%, and the standard deviation of Δz_0 is around +150% to -60%. It should be noted that this is greater than the estimated error in the retrieval associated with uncertainties in the Z-IWC relationship. Nonetheless, some clear trends are evident.

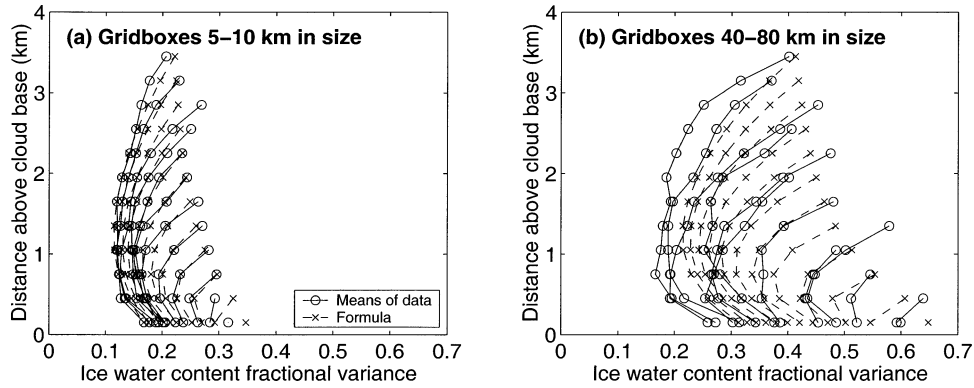


FIG. 8. (a), (b) The parameter f_{IWC} vs distance from cloud base z_{base} , for two different gridbox size ranges. Each line corresponds to clouds of a different depth, the solid lines being the means of the retrieved values, and the dashed lines being $0.28R$ in (a) and $0.52R$ in (b), where R is given by (15).

There seem to be approximate power laws relating both f_{IWC} and Δz_0 to d , while the dependence on s is closer to inverse exponential. Up to $d = 60$ km, the f_{IWC} data are quite well fitted by the following expression, shown by the dashed lines in Fig. 7a:

$$\log_{10} f_{IWC} = 0.3 \log_{10} d - 0.04s - 0.93, \quad (11)$$

where d is in kilometers and s is in meters per second per kilometer. At around $d = 60$ km, the observations suggest the beginnings of a scale break, beyond which mean f_{IWC} reaches an upper limit of around 0.4. Over 8000 events are present with $d > 60$ km, so this effect is believed to be statistically significant. It is therefore recommended that models with a horizontal gridbox size greater than 60 km use a value of $d = 60$ km in (11). The dashed lines in Fig. 7a become horizontal where this occurs.

The Δz_0 data are reasonably well fitted by the following expression, shown by the dashed lines in Fig. 7b:

$$\log_{10} \Delta z_0 = 0.3 \log_{10} d - 0.031s - 0.315, \quad (12)$$

where Δz_0 is in kilometers. This expression is valid for all values of d considered. It is interesting that for a horizontal gridbox size of less than 20 km, the mean decorrelation length is always less than 1 km and in many cases is less than the likely model vertical resolution in the middle and upper troposphere. Given that most ice clouds would be more than two layers thick, it may be a reasonable approximation for some high-resolution models to simply assume that ice cloud inhomogeneities are randomly overlapped (i.e., $\Delta z_0 = 0$).

We next consider the fractional variance and decorrelation length of the visible extinction coefficient, calculated from Z using (9). As explained in section 3, f_α is simply “scaled down” from f_{IWC} , according to the square of the ratio of the exponents in (9) and (8), yielding the following expression:

$$\log_{10} f_\alpha = 0.3 \log_{10} d - 0.04s - 0.96. \quad (13)$$

The decorrelation lengths are found to be exactly the same as for IWC.

In section 4b and Fig. 3d, an apparent dependence of f_{IWC} on distance from cloud top and base was found. Although the minimum in f_{IWC} coincided approximately with the maximum in shear, as would be predicted by (11), we would expect a constant shear to still result in a decrease of f_{IWC} with distance from cloud top, since the ice in the fallstreaks would get gradually more homogenized as it fell through the sheared region. To investigate this further, we consider only cases of shear between 0 and 10 m s⁻¹, and categorize each f_{IWC} observation according to its distance above cloud base z_{base} and its distance below cloud top z_{top} . Cloud boundaries are deemed to be where the cloud fraction falls below unity. The solid lines in Fig. 8 depict mean f_{IWC} versus z_{base} for clouds of thickness ranging from 0.3 to 3.6 km. Thicker clouds have a much noisier profile due to their less frequent occurrence in this dataset, so have not been plotted. Figures 8a and 8b correspond to gridbox size ranges of 5–10 and 40–80 km, respectively.

A clear tendency is evident for the fractional variance to decrease steadily from cloud top to perhaps half of its cloud-top value around two-thirds of the depth into the cloud, and then increase toward cloud base. Generally the fractional variance at cloud top is greater than that at cloud base. The minimum fractional variance of clouds 3.6 km thick is found to be around a third that of clouds 0.3 km thick. The $d^{0.3}$ dependence of f_{IWC} in (11) implies a factor of 1.87 difference in f_{IWC} between the data plotted in Figs. 8a and 8b, which is consistent with the observations. Nonetheless, the scatter for 40–80-km gridboxes is considerably more than that for 5–10-km gridboxes.

The dashed lines in Fig. 8 show the following fit to the data:

$$f_{IWC} = 0.153 \bar{d}^{0.3} R(z_{base}, z_{top}), \quad (14)$$

where

$$R = \exp[-1.5z_{\text{base}} - 0.2z_{\text{top}}] + \exp[-1.5z_{\text{top}}/(z_{\text{base}} + z_{\text{top}}) - 0.05z_{\text{base}}]. \quad (15)$$

The terms on the right-hand side of (15) may be explained as follows. The first term expresses the inverse exponential dependence of f_{IWC} on distance from cloud base, with the $-0.2z_{\text{top}}$ term in the exponent describing how the cloud-base value of f_{IWC} decreases with increasing cloud thickness. The second exponential expresses the inverse-exponential dependence of f_{IWC} on the *fraction* of the distance into the cloud from cloud top, with the $-0.05z_{\text{base}}$ term describing the decrease in the cloud-top value with increasing cloud thickness.

The full expression including dependence on gridbox size, wind shear, and distance from the cloud boundaries is then

$$\log_{10}f_{\text{IWC}} = 0.3 \log_{10}d + \log_{10}R - 0.04s - 0.66. \quad (16)$$

Similarly, for the fractional variance of visible extinction coefficient we have

$$\log_{10}f_{\alpha} = 0.3 \log_{10}d + \log_{10}R - 0.04s - 0.69. \quad (17)$$

As with (11) and (13), a maximum value of $d = 60$ km should be used in (16) and (17). No dependence of Δz_0 on the position in the cloud was found.

The fractional variance data were examined as a function of both temperature and absolute altitude, but no significant dependence was found. It should be noted that when the weak temperature dependence of the b coefficient in (7) is included in the retrieval of IWC from Z , this can feed through directly to a weak temperature dependence of f_{IWC} . However, since the temperature dependence of b is due to non-Rayleigh scattering of the larger particles at 94 GHz, rather than a meteorological phenomenon, and the fact that there are inherent uncertainties in our representation of non-Rayleigh effects, we are not confident that this represents a genuine temperature dependence of f_{IWC} . This finding is not at odds with the previous finding of a strong temperature dependence of *absolute* IWC (e.g., Stephens et al. 1990; McFarquhar and Heymsfield 1997), and certainly there is a robust temperature dependence of c in (7) for all radar frequencies (Liu and Illingworth 2000); but as discussed in section 3b, this does not affect the retrieval of f_{IWC} .

6. Conclusions

A large dataset of ice cloud observations by cloud radar has been used to derive essential information on cloud variability for use in the next generation of cloud schemes in operational numerical models. It is found that observed IWC distributions are usually well represented by lognormal or gamma distributions. Expressions have been fitted relating the fractional variances

of IWC and α to horizontal gridbox size, wind shear and distance above cloud base and below cloud top.

A clear tendency for increased shear to cause a homogenization of the cloud was evident, which resulted in significantly lower fractional variance. Due to the vertical distance over which the homogenization occurs, fractional variance tended to be greatest at cloud top and decrease with depth into the cloud, although beginning to increase again in the lowest third of the cloud.

In low shear conditions the vertical decorrelation length, which is independent of the parameter being correlated, was found to vary between 0.5 km for a 2-km gridbox to 2 km for a 300-km gridbox. Again, shear was found to reduce the correlation, principally because of the change in fallstreak orientation. Both fractional variance and vertical decorrelation length increased as gridbox size to the power of 0.3.

Some radiation schemes under development are able to represent the subgrid-scale variability of cloud properties, so would be able to make use of the results on fractional variance and vertical decorrelation reported in this paper. They typically split each column of gridboxes up into a number of narrower columns and perform radiative transfer calculations separately for each one. This way an arbitrary subgrid-scale distribution of cloud properties may be specified, and partially cloudy gridboxes treated in a more consistent way. To run the full radiation scheme on each subcolumn would be very computationally expensive, but by simply running each different wave band of the radiation scheme over a different subcolumn, no significant increase in cost is incurred (H. W. Barker 2002, personal communication).

It is not immediately clear how these results should be applied in partially cloudy model gridboxes. We excluded partially cloudy gridboxes from the analysis because to include them would imply that the cloud-free regions were distributed evenly through the gridbox, in the same way as in fields of cumulus or broken stratocumulus. In the case of ice clouds, partially cloudy gridboxes are invariably those containing the edges of a large cloud that may span many gridboxes in the horizontal. Therefore, the best approach for a model may be to retain a parameterization for cloud fraction, and apply the formula for fractional variance to only the cloudy part of the box, using a value of d that reflects the size of the cloudy sector of the box, rather than the size of the gridbox as a whole.

The large majority of the clouds considered in this study would have been nucleated from cloud top in small generating cells, so it would be useful to extend this study to the Tropics where most ice clouds are anvil cirrus. It should also be pointed out that the relationships derived here apply principally to stratiform clouds, and model gridboxes containing storm cells penetrating into more extensive ice cloud should treat the overlap of the convective and stratiform regions separately, with the convective component presumably conforming to maximum overlap. The forthcoming National Aeronautics

and Space Administration (NASA) CloudSat mission involves launching a 94-GHz radar in space in 2004 and will be able to provide data on IWC variability and overlap on a global scale.

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